Engaging and effective teaching and learning in secondary school mathematics and statistics

School resources

Mathematics and statistics contribute powerful tools for finding solutions to the world's societal, environmental, humanitarian, and economic challenges. Mathematics and statistics are essential for fighting inequity, demanding social justice, saving our environment, and developing collaboration and understanding between people. Successful secondary mathematics and statistics learning is highly empowering. It gives students options for continued study and employment. It helps them interpret, understand, and think critically about mathematical ideas and enables them to engage in society. How secondary school mathematics and statistics (subsequently collectively referred to as mathematics) is taught can make an important difference for students and what they will do in their lives with and for others. This guide presents ideas about what makes for effective and inspiring secondary school mathematics teaching and learning, with a particular focus on Aotearoa New Zealand.

Inspiring and effective secondary school mathematics teaching deftly and deliberately weaves together many areas of knowledge for teaching. Teachers need to know themselves and their learners well, their mathematics content and its history and real-life applications, how to assist students to recognise and move past partial mathematical understandings, and how to set up motivating investigative and discussion-based learning experiences that develop understanding and retention. Teachers need to convey their own joy in mathematical endeavour and nourish such joy in students. They need to know how to help students see themselves as competent, confident, and excited learners of mathematics and how to develop their thirst for more and deeper mathematical understandings. By teaching in these ways, teachers engage, uplift, and empower students to be confident, capable, and enthusiastic doers and users of mathematics.

This guide is focused around seven areas, which are directly related to students:

- Meeting the needs and interests of all
- Motivating mathematical learners
- Making meaning and developing understanding, retention, and fluency
- Managing mathematical perseverance and satisfaction
- Multiple approaches to mathematics teaching and learning
- Making connections with and between mathematical ideas
- Making sure students see themselves as doers and learners of mathematics and statistics

The discussion within each area is guided by a whakaaro – a thought, kīwaha, saying, proverb or whakatauki. Key implications for practice and bullet points signal suitable teacher actions. Each finishes with a discussion of how teachers might know if they are successfully implementing this area of secondary mathematics teaching.

Meeting the needs and interests of everyone

He aha te mea nui o te ao? He tangata, he tangata, he tangata[1].

Just as this whakatauki suggests, the discussion must begin with considerations of the people involved in teaching and learning and their collective whakapapa and backgrounds: teachers, who they are teaching, the people important to students, and those contributing to decisions about how learning happens[2]. Everyone brings their own worldview to their interactions and teaching, and their own ways of experiencing joy in mathematical endeavour. Teachers need to recognise that those they work with and for may hold different worldviews and ways of thinking, learning, and feeling from theirs.

Meeting the needs and interests of everyone requires teachers to understand each person they work with, what they care about, what they know and do not know, can and cannot do, what their beliefs and interests are, how they feel about mathematics as a subject, how they learn well, and what motivates them to know and learn more. Understanding others requires teachers to have knowledge of who they themselves are, what they bring to the relationship, what they care about, what they already know and do not know, can and cannot do, what their beliefs and interests are, how they learn, teach, communicate, and work in partnerships with those important to their students, and what motivates them to teach better.

Learning experiences that can help teachers develop such understandings while keeping their focus on mathematical ideas include those that link to everyday real life open activities, in which students can investigate, make mathematical decisions, discuss, and debate. For such learning experiences, teachers need to:

- Know students well, who is important to them, and what they already know.
- Know how to shape mathematics learning experiences so that students can be themselves and can understand how their learning relates to what is important to them.

Teachers will know they are successfully meeting everyone's needs and interests when students are keen to greet and interact with them, and when they see students enthusiastic and confident about their mathematics learning and progress, and keen to share what they know about mathematics and their other passions.

Motivating mathematical learners

Kei te ora pai tātou katoa.

Tā Mason Durie's whare tapa whā [3] helps teachers understand that there are interrelated dimensions that contribute to our health and wellbeing. The model highlights the balance needed between all aspects of our wellbeing for us to be able to thrive - cognitive and emotional (taha hinengaro), physical (taha tinana), social and family (tata whānau), and spritual wellbeing (taha wairua) and our grounding and roots (whenua). Ideally, in mathematics teaching teachers want to nourish all dimensions of wellbeing for each student to help maximise their learning. Students' motivation to learn mathematics can only be enhanced if they feel well, well-supported, cognitively engaged, socially connected, able to be themselves, and able to flourish as they learn, interact, and achieve[4]. Motivation for learning mathematics is also linked to students experiencing autonomy, relatedness, and competence[5]. Having agency over mathematical decision making (for example, within learning tasks and how they are undertaken) enables student autonomy. Opportunities to discuss mathematical ideas with others and see how these ideas are present outside the classroom contribute to students' understanding and sense of relatedness in mathematics learning. Students being able to experience success and achievement and to know what they have learnt and achieved enhances their feelings of mathematical competence.

Learning experiences that can help teachers attend to students' health, wellbeing, and motivation while keeping their focus on mathematical ideas include exploring mathematics concepts outside the classroom, embodied learning experiences such as making the shapes of algebra, geometry, and measurement topics with their hands, arms, and bodies (attending to ideas of physical wellbeing – te taha tīnana and relatedness), and ensuring students can confer and discuss mathematical ideas with each other, their family and whānau, teachers and kaiako (enhancing social wellbeing – te taha whānau and relatedness). They include helping students feel challenged, resilient, successful, and acknowledged (enhancing spiritual wellbeing - te taha wairua and feelings of competence), and helping them enjoy rich mathematical learning experiences (ensuring opportunities for cognitive and emotional wellbeing - te taha hinengaro and student autonomy). To help maximise students' motivation to learn mathematics, teachers need to:

- Hold and demonstrate high expectations of all.
- Plan for diverse learning experiences that cater for differences in learning readiness and needs, which support students in all dimensions of he whare tapa whā, autonomy, relatedness, and competence.
- Prioritise developing students' motivation for learning mathematics through providing engaging, challenging tasks, and warmly expecting and celebrating their success.
- Highlight contexts where mathematics can help resolve problems in students' everyday lives and wider worlds.

Teachers will know they are successfully motivating students when students are keen to get to their learning, interested to know what they will be doing and learning, able to set and monitor their own mathemtics learning goals and progress, and when they are persistent and resilient in their mathematical undertakings.

Making meaning and developing understanding, retention, and fluency

He rangi tā matawhāiti, he rangi tā matawhānui[6].

To engage in mathematics, students need to develop understanding of and the ability to retain new ideas, concepts, and tools[7] so they can identify which idea is useful when, and have fluency in using and adapting these ideas to contexts they may meet. It needs to be clear to students that their teacher expects them to understand the ideas presented, to swiftly recall these, and to be able to apply them in familiar and unfamiliar problems and ways. Meaning making is enhanced when teachers identify and plan according to students' prior mathematics learning and needs specific to the content at hand, arrange worthwhile mathematical learning tasks, and strategically build on students' thinking[8]. Students arrive with varying strengths and gaps in mathematical understanding. Differentiated tasks are often needed to cater for such differences.

Learning experiences that can help teachers develop students' mathematical understanding include using equipment, diagrams, representations, discussion, and worked examples to explore the focus concept. Frequent opportuntities for students to revisit and practise key skills (such as manipulating algebraic equations, using fractions, using integers, rounding, and using measuring scales and units) is important for assisting with retention and fluency. Learning experiences that can assist involve repetition of key ideas, rules, and procedures (such as textbook, worksheet, or online questions), exploratory tasks investigating patterns and relationships which enable students to use these key ideas in unfamiliar contexts, close exercises (such as worked notes or examples with gaps for students to fill), games

in which key processes can be practised many times in a fun and challenging way, and cooperative activities which involve language use and discussion of key mathematical ideas. Further ways to assist with developing retention and fluency include students writing and participating in topic-based raps, songs, and action songs, creating mnemonics, notes, diagrams, story-telling, and the teacher using surprise, as these types of lesson activities engage creativity and pleasure while reinforcing ideas. To help students make meaning of and remember mathematical ideas, teachers need to:

- Expect all students to understand simple and complex mathematical ideas and know how to support students with this.
- Expect all students to remember and be proficient in using mathematical understanding, skills, and tools, and know how to support them with these.

Teachers will know they are successfully helping students make meaning and develop understanding when they see confusion replaced by comfort, when they see students increasingly capable with mathematical language and explaining their thinking, and when they hear comments like 'I get this', 'that makes sense', and 'I didn't get it before, but now I can do this'. Teachers will know they have been succesful in helping students remember and use new mathematics learning when they recall key ideas when needed and when they think to use these ideas to interrogate mathematical situations and solve problems.

Managing mathematical perseverance and satisfaction

*Kia **āta** tupato me karawhiua!*

Partially formed mathematical understandings can support or get in the way of mathematics learning. Many students have a collection of partial understandings for teachers to find and accommodate[9]. Teachers must know about common mathematical misconceptions and errors in order to set up ways for students to realise they need to gain further understanding. Teachers need to prepare students that, as individuals and as a group, they will have partial mathematical understandings to build on and some errors to find and fix – all learners do – and that part of learning mathematics is to identify these and use them to help everyone learn more and learn better.

Learning environments need to be places where trying, making progress, making mistakes, and persevering are seen as opportunities for learning – as useful rather than as shameful or annoying. Learning experiences that can help students experience the satisfaction of moving past partial understandings include setting up cognitive conflict situations for discussion, such as by presenting carefully chosen alternatives that expose a misconception for debate (for example, some think that ½ + 1/3 = 2/5 and others think ½ + 1/3 = 5/6. Who is right and why, and what is a real world example that you could use to convince someone else about your thinking?). Other learning experiences that can help students feel satisfaction, pleasure, and joy from making progress with learning they find challenging include using hands-on activities, equipment, guided discovery, investigations, problem solving, and using representations to explore mathematical ideas [10]. These styles of activities help lead to student satisfaction and enjoyment because they promote understanding and they tap into more aspects of students' wellbeing than cognitively focused tasks can (for example, they promote physical wellbeing through movement, cognitive and emotional wellbeing through students having autonomy and being able to work things out for themselves through problem solving, and social wellbeing through working with others). To help students develop mathematical understanding and perseverance, teachers need to:

- Use learning experiences in which students can develop individual and collaborative problem solving and resilience, achieve success, and acknowledge one another's success (such as cooperative activities, paired investigations, whole class investigations).
- Anticipate commonly held partial understandings, misconceptions, and errors (for example, through talking with other teachers, searching online, and reading to find these for the content they are teaching, as well as ways to help students avoid and overcome these).
- Know how to help students be ready and keen to identify and learn from partial understandings they bring (such as helping students realise these are usual for all students and to be expected, having students expect cognitive conflict based questions and pre-topic assessments, and knowing their teacher will be using these types of tools to inform their teaching decisions).
- Know how to help students develop mathematical understanding in diverse ways (using equipment, diagrams, discussion, representations, experiments, interactives, and cognitive conflict).

Teachers will know they are succesfully supporting students' mathematical perseverance and satisfaction when they see them persist with a problem despite experiencing difficulty, when they hear them confidently discussing differing mathematical ideas, when they are comfortable to let teachers know when they have an answer different from others, and when they reach for equipment, representations, or other tools to help interrogate mathematical thinking.

Multiple approaches to mathematics teaching and learning

E koekoe te tūī, e ketekete te kākā, e kūkū te kererū[11].

For many people, mathematics learning followed a fairly consistent pattern of teacher explanation, teacher modelling of worked examples, copying notes and examples, then doing practice questions from textbooks. This approach may have worked for many, but it has not worked for all. Unwavering routines in mathematics learning can lead to boredom and beliefs that mathematics is a set of rules and procedures[12]. Teachers need everyone to be successful mathematics learners so they need to teach in diverse, inspiring, interesting, and engaging ways.

Varying the classroom routine (for example, starting with a game that leads to new mathematical thinking), physical space (such as different desk set ups, learning outside the classroom), and the ways of working (individual, pairs, groups) can help lead to learning experiences with variety that cater for diversity. Using digital technology to explore fascinating facts and interesting frequencies and trends (statistical displays, reports, and data sets), using hands on investigations (for example, to explore maximum or minimum values for specific measurement tasks), and performance, dance, drama, and students' languages (such as demonstrating transformation geometry ideas in sāsā, students using mathematics in taking the role of expert in a social or environmental scenario, collecting and using mathematical terms from students' languages in classroom displays) can all help link new learning to what students already know, enjoy, and care about[13]. Such teaching helps students explore different ways of solving problems and explaining mathematical thinking, and helps show that all approaches can add to learners' understandings, discussions, and achievement. Teachers need to:

- Understand that learning happens in many ways and know many ways to teach.
- Use diverse opportunties for students to learn in every lesson.

Teachers will know they are successfully encouraging diverse approaches to mathematics learning when their lessons are vibrant, energised, and exciting, and when they see students using varied methods and strategies to explore, explain, discuss, and solve mathematical problems.

Making connections in, with, and between mathematical ideas

Ano he whare pungawerewere pai.

While many curricula and long term teaching plans present mathematics in separate topic areas, there are many connections between different aspects of mathemtics, in both real life and mathematical contexts. For example, in creating a feast, organising a sports team's trip, or building a house, ideas of measurement, number, geometry, statistics, and algebra can all be at play. If we focus in on mathematical contexts, we see further connections – we realise that exploring algebraic patterns helps us understand how numbers work, that measurement, number and algebra are connected through the rules that help us calculate areas and volumes, and that working out probability values can draw from number, algebra, statistics, and geometry, depending on the problem, for example. A teacher's role is to support students to appreciate such interconnectedness of mathematics so that they don't see mathematics as a collection of separate topics. They can do this by exposing connections between various ways of solving problems, between representations, and between mathematics learning and real life[14]. Students seeing a broad picture, the many purposes of mathematics, and how each day's learning contributes to these, can also help them connect mathematical ideas[15] and expect that such links exist and can be useful for being able to transfer what is learnt in class to situations they meet out of class. Such connections can help demonstrate recognition of and respect for Indigenous knowledge and ways of being and doing, and ensure that bodies of both Indigenous and mathematical knowledge are given acknowledgement, respect, perspective, and time[16]. For example, exploring Western mathematical ideas that may be visible in te ao Māori, and attending with integrity to what can be learnt about te ao Māori as well as to Western mathematics in the process, can assist with understanding how both knowledges may help us interpret and describe ideas.

Learning experiences that can help students develop mathematical connections include ensuring links are made within every lesson between the prior learning, new learning, and mathematical and other applications of the new ideas. Teachers can also emphasise mathematical connectedness by demonstrating and encouraging mathematical argumentation (including proof), encouraging discussion of links between ideas explored in different ways, and ensuring students can use multiple examples and representations to communicate mathematical ideas (such as context, table, graph, explanation, and rule). Teachers need to:

- Know how and why mathematical ideas connect within and across topics and how to help students appreciate and use these connections.
- Use activities that enable students to explore connections between mathematical ideas, tools for understanding and explaining, and experiences outside the classroom.

Teachers will know they are successful in helping students see and use mathematical connections when they hear them discussing mathematics using statements like 'this is like we did in our last topic', 'this is another way of showing the information', 'we use this in my sport to work out player stats', and 'mum uses really accurate measuring tools like this for doing her quotes'.

Making sure students see themselves as doers and learners of mathematics and statistics

Ko ahau, ko koe, ko tātou.

Students are more likely to see themselves as doers and learners of mathematics when they frequently see mathematics as a human endeavour[17]. Helping students understand that mathematical ideas have been created to solve problems, explain, develop, protect, and predict the world around them by many people over many generations across many parts of the world is a teacher's responsibility and pleasure. There are fascinating stories to share about the whakapapa and background of mathematical ideas, tools, and techniques. Such story-telling helps students engage with and remember mathematical ideas and understand why teachers are sharing specific mathematical ideas with them[18].

Learning experiences that can help students see themselves as doers of mathematics include story-telling, students investigating historical and current perspectives and applications of mathematics content, thematic topics, tasks that link to contexts around the world[19], and exploring mathematics used in other curriculum areas, hobbies, sports, and pastimes. Teachers need to:

- Know who was involved and why and where mathematics ideas originated.
- Share information about mathematicians and the history of mathematics.
- Discuss who in the community uses mathematics and why and how they use it.

Teachers will know they are successfully helping students see themselves as doers of mathematics when they are interested in the history of mathematical thinking, ask questions about where ideas came from, who developed them, why they are used, and when they opt into further mathematical study.

Endnotes

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