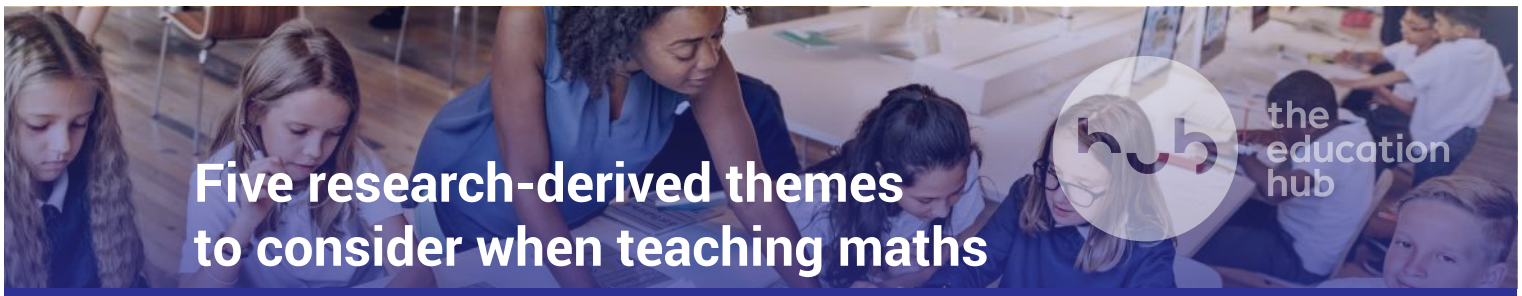

Five research-derived themes to consider when teaching maths

School resources

The task of teachers of primary mathematics is to bring important mathematics concepts and skills together with students' attitudes, beliefs, prior knowledge and thinking – and to do so in dynamic learning spaces. Bearing in mind the complexities of each of these three things on their own, it is clear that what works for one concept or skill, one group of students, or one place might not work for a different idea, different students or a different place. Faced with this complexity, there is a strong urge to simplify. This is why there are so many 'silver bullet' solutions proposed in mathematics teaching and learning, and teachers can feel like they are constantly on a pendulum swinging from one approach to another. The constant stream of advice, and the public arguments about what is 'best', are exhausting for everyone involved and have not yet resulted in the outcomes we want for all our students. So where should teachers put their mathematics-teaching energy in order to make things better?

Teachers design and implement learning experiences, so thinking about planning and teaching mathematics as providing **opportunities to learn** can be helpful for finding a way forward through all the advice and complexity. Each time a teacher creates a learning experience, be it giving an explanation, posing a problem to solve, giving an example to try, or allowing time on a mathematics computer learning programme, an opportunity for learning is opened. The teacher decides what content is in, and what is out. They decide in what mode information will be presented and what mode students' responses will take. Each of these decisions is critical in shaping the opportunity to learn. An international research team have a nice phrase for this: the explanations and learning experiences teachers provide define 'what is made possible to learn'[1]. No matter what form the opportunity to learn takes (its shape will depend on the mathematics idea, the students and the context), here are five key things to consider when trying to make 'what is possible to learn' as valuable and accessible as possible.

Talk

There is a lot of evidence that talk is important to mathematics learning[2]. Stereotypical mathematics classes are portrayed as silent, with rows of students completing examples. Some quiet, individual work in mathematics is useful and important, but talk seems to be one way to leverage student progress in mathematics. Talk as a theme embraces a range of aspects: it is important in terms of what the teacher says (how things are explained, what questions are asked); it is important that students get to talk to each other and discuss their thinking, explaining and justifying their ideas; and it is important who gets to talk (does everyone have a chance to speak?). It may be an oversimplification, but a useful thing to consider is that 'the person doing the talking is doing the learning'. We might think that the listener is the student, but through organising their ideas, accessing and using the necessary vocabulary to explain their ideas, and engaging with the lesson enough to offer ideas out loud, it is actually the speaker who is learning. Listeners also learn, but talking is important.

To embrace this theme when designing opportunities to learn, consider:

- How you introduce and support the use of mathematical language
- How to get your students talking productively
- How you can make sure everyone has a chance to talk
- How you can change the balance of teacher-talk to student-talk so it favours the students

Tasks

The tasks that are given to students are central in defining 'what is possible to learn'[3]. If students are individually colouring in triangle shapes on a worksheet, they will have different possibilities to a group of students with a bucket of regular and irregular shapes that they are sorting into 'triangles and not triangles'. These two tasks target the same understandings, but open different opportunities to learn. Tasks need to be a good match for teachers' intentions. Sometimes it is necessary to target specific skills or concepts; although it is important to take care that the tasks given across the span of a week or a term do not convey the message that mathematics is just about skills practice. To promote engagement and powerful learning, the tasks teachers choose should show that mathematics is also for solving puzzles and problems, working things out and making connections[4].

Mathematics researchers coin various terms for the types of tasks that have a lot of valuable mathematics contained inside them, such as open tasks, rich tasks, group-worthy tasks, and authentic tasks. Some tasks start small but can be taken a long way mathematically. This is often the case for tasks that involve patterns because they can be extended to generalisations. The phrase 'low floor - high ceiling' refers to tasks like this – everyone can make a start because the entry point is easy, and students can take the mathematics as far as they like because the ceiling is high. These tasks are not easy to invent, but there are lots of places to find them[5]. Simple modifications can enrich a task; for example, instead of a worksheet of addition pairs to ten, pose the question '10 is the answer, what might the questions be?', and then examine the patterns that emerge. Finally, the tasks teachers choose give messages about what mathematics is. Worksheets with repetitive exercises create a very different message to a problem-solving task that relates to students' background and interests.

To embrace this theme when designing learning opportunities, consider:

- Does your task match your intentions (for example, if you want exploration of ideas, have you given a task where there are ideas to explore)?
- Can the tasks you use be 'flipped' or modified to open them up to more possibilities for students?
- What messages about what mathematics is and what it means to 'do mathematics' are embedded in the tasks you set?

Tools

Tools for learning mathematics have always been important in primary school. Different ways of representing numbers and ideas can form a bridge for students between things they can explore in the 'real world' and abstract manipulation of ideas in their heads. Some representations are good for showing some ideas, but less good for showing others. For example, a collection of buttons is good for showing simple amounts like '6' or '3', but is not so good for showing '76' (which gets a bit messy and you can't really tell it's 76) and is no good at all for showing 4 ½. Having students draw regions and then find a fraction of it often leads to rough drawings of unequal parts, which does not help conceptual development. Using fraction circles, fraction tiles or fraction walls can support the understanding that fractions are the result of dividing into equal pieces.

When we choose tools we need to choose carefully, so that they provide the best possible model for the idea we are exploring. We also need to use a variety of tools so we can present the idea to students in different ways[6]. Each different representation opens up a new opportunity to learn that might capture someone who couldn't learn from previous representations. Some tools are very powerful and useful for lots of ideas; mastering how they work helps students with several mathematics concepts. A key tool is

the number line; this way of modelling the magnitude of numbers helps students develop flexible number sense – especially with fractions.

To embrace this theme when designing learning opportunities, consider:

- What opportunities do my students have to interact with tools and representations in their mathematics learning?
- How many of these are hands-on opportunities?
- How could I introduce several different ways of modelling an idea using tools or representations, to make the idea more accessible?

Thoughts

This theme has two components: the teacher's thoughts and the students' thoughts[7]. What teachers think about mathematics, and what they think about their students, are key determiners of the progress that students will make. Seeing them as capable, taking an asset-based approach, and regarding mathematics as valuable and fun will set teachers up to design learning opportunities that will help all students progress.

Students need to develop a mathematics learner identity that supports positive relationships with mathematics. This might include seeing mathematics as relevant and valuable for them, seeing themselves as capable and resilient learners, feeling comfortable when they are 'doing mathematics', embracing challenge, and enjoying mathematics activities. The relationships students develop with mathematics is also a key determiner of their progress, especially in later primary years.

To embrace this theme when designing learning opportunities, consider:

- What messages do my choices – about what mathematics we do and how we do it – convey to my students about how I see mathematics?
- What messages do I give my students about my view of them as capable mathematics learners?
- What opportunities do I give learners to explicitly build positive mathematics learner identities, and to talk and think about themselves and their relationship to mathematics and mathematics learning?

Together

The final theme, 'together', expresses the idea that mathematics is for everyone. Mathematics has traditionally been viewed as being for an elite few 'geniuses' who 'get it'. It has also been framed as being more for particular groups of people than for others. People who do mathematics well are 'smart', people who struggle with mathematics ideas are 'dumb'. When these ideas are written down or talked about out loud, it is easy to see and hear how harmful they are, and yet they pervade our subconscious. Many people wear 'I'm no good at maths' as a badge of honour, rejecting what they think mathematics stands for.

Because mathematics is essential for living an adult life in the twenty-first century, everyone needs to have access to it. Mathematics is often the gatekeeper to jobs and further study, making it a matter of equity. When teachers design opportunities to learn, it is important to think about how to include everyone and make the ideas as accessible as possible. 'Together' has a further meaning: through collaboration and discussion with colleagues, students and families/whānau, teachers can grow their mathematics teaching practice. It is a big job, so help each other.

To embrace this theme when designing learning opportunities, consider:

- What might be the consequences of how I decide how to approach this idea with my students?
- How might prior experience impact engagement with my learning experience? How can I respond to that?
- Can all my students 'see themselves' in the learning opportunities I am designing for them?

So, how should teachers choose what to do when teaching mathematics? When making choices, it is important to consider:

- The mathematics you are teaching
- The students you have
- The spaces and resources available to you

What works for the combination of these things will differ by idea, by student, by teacher and by school. A teacher's job is to create learning opportunities that open up the mathematics to the students as well as possible in the spaces where they work. If teachers regard each decision about what students will do as framing 'what is possible to learn', it will help to choose and design learning experiences carefully. Use information from the students' responses to decide whether to do more of something, or less; or to try something in a new way. When designing opportunities to learn, consider talk, tasks, tools, thoughts and togetherness. Teaching mathematics is not straightforward, but it is incredibly important and rewarding.

The authors would like to thank Ellen Benes and Megan Clune for their contribution to this guide.

Recommended further reading

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Endnotes

1 Kullberg, Kempe & Marton, 2017.

2 Hunter et al., 2020; Hunter et al., 2018; Walshaw & Anthony, 2008.

3 Kullberg, Kempe & Marton, 2017.

4 Daragh, 2021; Hunter et al., 2020; Foster, 2018.

5 https://nzmaths.co.nz/problem-solving
https://nrich.maths.org/8769
https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

6 Hunter et al., 2020.

7 Hunter, Hunter, & Anthony, 2020.

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